Pulley Friction: Three-Dimensional Analysis of Cable Forces in a Suspended Mass System

Anne Krikorian

First Monday Late Lab Group:
Jesse Bertrand, Ryan Carmichael, Anne Krikorian, Noah Marks, Ann Murray

E6 Lab Report
February 29, 2008

Abstract:
We computed the experimental average coefficient of friction between a pulley axle and a pulley sheave in a system of three pulleys and four suspended weights in static equilibrium. The pulleys were suspended from the ceiling in a triangular arrangement, with the cords running over the pulleys attached in a knot with a constant suspended weight, while we altered the weights hanging from the other ends of the cords. After allowing the system to settle into an initial equilibrium state, we then manually found the equilibrium from above the equilibrium point and from below. We measured the location of the knot and the pulleys in three-space, as the difference in the location of the central knot changed due to the friction of the pulleys. Our calculated averages for the coefficients of friction of pulleys 1, 2, and 3 were .03515±.2, .01582±.2, and .0469±.2, respectively.
Purpose:

We will measure the coefficient of static friction between the axle and the sheave of a pulley by analyzing a three-dimensional system of forces.

Theory:

Pulleys are often assumed ideal, or massless and frictionless, in order to simplify analysis of pulley systems. However, in real applications, pulleys have both mass and friction. While in the system analyzed in this laboratory the mass is insignificant, the friction between the axle and the sheave does affect our analysis.

The friction in a pulley comes from the natural roughness of the materials used and any flaws in the surfaces of the axle or sheave. The friction then yields a force that opposes motion. By repeating many trials (i.e., many different suspended weights), we calculate the average coefficient of static friction, which reduces our uncertainty. Taking measurements from above and below the equilibrium point also aids in increasing the accuracy of our findings.

This model of the pulley, with friction included, results in the following free body diagram:
Our analysis is based on the system being in static equilibrium, which means that all forces must sum to zero. Thus, $\Sigma F = 0$ and $\Sigma Mo = 0$. In our calculations, we sum the moments around the pulley axle, having determined the tensions in the cables, to find the value of the force of friction on the pulley. To further improve our calculations, we first correct our coordinates to more closely determine the point of concurrency of the rope on the pulley. While this correction yields only a slight alteration to our result, that alteration does bring us closer to the true value of the coefficient of friction.
This figure details the mathematical process of correcting the point of concurrency.

The mathematical process itself is detailed below:

\[ P_{2\text{ new}} = P_2 - P_1 \]
\[ P_{3\text{ new}} = P_3 - P_1 \]
\[ P_{4\text{ new}} = P_3 - P_1 \]
\[ P_{2\text{ final}} = (P_{2\text{ new}} \times [1,1,0]) + [0,0,D] \]

Where D is
\[ D = \sqrt{x_2^2 + y_2^2} \tan(\theta + \varphi) \]

When \( \theta \) and \( \varphi \) are
\[ \theta = \tan^{-1} \left( \frac{z_2}{\sqrt{x_2^2 + y_2^2} - r_{\text{sheave}}^2} \right) \]
\[ \varphi = \sin^{-1} \left( \frac{r_{\text{sheave}}}{\sqrt{x_2^2 + y_2^2} - r_{\text{sheave}}^2} \right) \]

\[ \hat{e}_{P_2} = \frac{P_{2\text{ final}}}{\|P_{2\text{ final}}\|} \]

\[
\begin{bmatrix}
0 \\
0 \\
W1
\end{bmatrix} \begin{bmatrix}
\hat{e}_{P_{2x}} & \hat{e}_{P_{3x}} & \hat{e}_{P_{4x}} \\
\hat{e}_{P_{2y}} & \hat{e}_{P_{3y}} & \hat{e}_{P_{4y}} \\
\hat{e}_{P_{2z}} & \hat{e}_{P_{3z}} & \hat{e}_{P_{z}}
\end{bmatrix}^{-1} = \begin{bmatrix}
T_2 \\
T_3 \\
T_4
\end{bmatrix}
\]
Procedure:
We began by measuring the location of each of the pulleys. With this data recorded, we proceeded to load the ends of the cords with weights. At this point we checked to see that the pulleys were in line with the cords, and if they were not corrected this. For each of our three weight combinations, we allowed the system to settle into equilibrium first from above the equilibrium point, and then from below the equilibrium point. When the system settled, we used a laser pointer attached to the middle knot point and projecting on the floor to determine the location in the x-y plane of the knot, and measured the z location with a surveyor’s tape.

Apparatus:
Results:

<table>
<thead>
<tr>
<th>Group</th>
<th>Avg. $\mu_s$ for pulley 2</th>
<th>Avg. $\mu_s$ for pulley 3</th>
<th>Avg. $\mu_s$ for pulley 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ME</td>
<td>0.081012 ± .2</td>
<td>0.05542 ± .2</td>
<td>0.043314 ± .2</td>
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<tr>
<td>ALL</td>
<td>0.03515 ± .2</td>
<td>0.01582 ± .2</td>
<td>0.0469 ± .2</td>
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</table>

Discussion:

These results show trivial values for the coefficients of static friction of the pulleys. The significance of these findings is that they show that the frictionless pulley assumption is valid for this situation; the frictions do not in fact have a strong effect on the outcome of our results, as seen by the incredibly large uncertainties (compared to the
values). It is possible that, with more accurate measurements, we would determine that
the frictionless pulley assumption is not value. However, with error from the flex of the
surveyor’s tape in measuring z-coordinates, the erratic behavior of the laser pointer used
to measure the x- and y-coordinates, the pulleys not being aligned with the cords, and
other possible causes of error (not excluding the difference in static friction when
measured from above as compared to below), the results we receive are trivial. Also to be
noted is that, within our error ranges, is the physical impossibility of a negative
coefficient of static friction: that is, friction that contributes to instead of resists
movement.

**Conclusion:**

In conclusion, while our calculations did yield values for the coefficients of static
friction, the results were insignificant when compared to the uncertainties. This signifies
that the friction of the pulleys is negligible when the measurements have as much error as
ours do. The implication is that our data collection techniques are unsophisticated enough
that the more sophisticated model of the system (that which includes friction) does not
improve our results. In future trials of this lab, it might be beneficial to improve our
techniques in measuring the coordinates of the knot-point and in aligning the pulleys with
the cords.

**References:**

Everbach, E. Carr.


‘Laboratory on Concurrent Force Equilibrium in 3-D.’ Swarthmore College,
2008.

Appendices are the joint work of Ryan Carmichael and Anne Krikorian, except
where noted otherwise.

All figures are the creation of Michael Ticehurst, Swarthmore class of 2011,
except that in Appendix B, which is the creation of Zachary Eichenwald, Swarthmore
class of 2010.
List of Appendices:

A.......................................................... Table of Mu Values
B.......................................................... Diagram of Pulley System
C......................................................... Raw Data: First Monday Late Lab Group
D.......................................................... Raw Data: Entire Class
E.......................................................... Matlab Script
## Appendix A:

<table>
<thead>
<tr>
<th>Group</th>
<th>Trial</th>
<th>Cw $\mu_s$ for pulley 2</th>
<th>Ccw $\mu_s$ for pulley 2</th>
<th>Cw $\mu_s$ for pulley 3</th>
<th>Ccw $\mu_s$ for pulley 3</th>
<th>Cw $\mu_s$ for pulley 4</th>
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* Cw = clockwise, Ccw= counterclockwise
Appendix C:

2/4/2008  First Monday Late Group Data for Lab #1
Group Members: Jesse Bertrand, Ryan Carmichael, Anne Krikorian, Noah Marks, Ann Murray

Note: all point measurements are in inches.

P2=(-54.25, 0, 119); P3=(54.25, 0, 119); P4=(0, , 119)

Test 1:
- W2= 524.38g; W3= 805.14g; W4= 969.43g
  P1(above)= (9.375, 57.5, 78.0625); P1(below)= (8.875, 58.25, 75.8125)

Test 2:
- W2= 341.41g; W3= 988.11g; W4= 969.43g
  P1(above)= (21.5, 51.625, 80.625); P1(below)= (19.75, 52.5, 78.4375)

Test 3:
- W2= 606.33g; W3= 988.11g; W4= 704.51g
  P1(above)= (19.75, 26.625, 87.1875); P1(below)= (18.125, 27.875, 85)

Note: the z adjustment has been accounted for.
## Appendix D:

Datasheet for 3DForce Lab
Spring 2008

Tabulation Sheet

**Support Locations:**

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Appendix E:

%Program written by Ryan Carmichael and Anne Krikorian with aid from D. Kao

load ThreeDforces
for index = 1:18
    %coordinates in inches relative to the original origin
    P1US = coordsfrombelow (index,:);
    P2US = [-54.25, 0, 119];
    P3US = [54.25, 0, 119];
    P4US = [0, 156.125, 119];

    %coordinates in meters relative to the original origin
    P1SI = P1US * 0.0254;
    P2SI = P2US * 0.0254;
    P3SI = P3US * 0.0254;
    P4SI = P4US * 0.0254;

    %new coordinates after making P1 the orgin
    newp1 = [0,0,0];
    newp2 = P2SI - P1SI;
    newp3 = P3SI - P1SI;
    newp4 = P4SI - P1SI;

    %masses in grams
    M1 = grams(index,1);
    M2 = grams(index,2);
    M3 = grams(index,3);
    M4 = grams(index,4);

    %converting mass to newtons
    W1 = M1/1000 * 9.81;
    W2 = M2/1000 * 9.81;
    W3 = M3/1000 * 9.81;
    W4 = M4/1000 * 9.81;

    %radius of axle in m
    r1 = 0.0047625;

    %outer radius of the pulley in m
    r2 = 0.01905;

    %projection of suspension cord
    ds2 = sqrt(newp2(1)^2 + (newp2(2))^2);
    ds3 = sqrt(newp3(1)^2 + (newp3(2))^2);
    ds4 = sqrt(newp4(1)^2 + (newp4(2))^2);
%auxiliary distance
AD2 = sqrt((ds2 - r2)^2 + (newp2(3))^2);
AD3 = sqrt((ds3 - r2)^2 + (newp3(3))^2);
AD4 = sqrt((ds4 - r2)^2 + (newp4(3))^2);

%first vertical angle
FVertAng2 = atan((newp2(3))/(ds2 - r2));
FVertAng3 = atan((newp3(3))/(ds3 - r2));
FVertAng4 = atan((newp4(3))/(ds4 - r2));

%second Vertical Angle
SecVertAng2 = atan((newp2(3))/(ds2 - r2));
SecVertAng3 = atan((newp3(3))/(ds3 - r2));
SecVertAng4 = atan((newp4(3))/(ds4 - r2));

%incremental angle
IAng2 = asin(r2 / AD2);
IAng3 = asin(r2 / AD3);
IAng4 = asin(r2 / AD4);

%final vertical angle
FinVertAng2 = FVertAng2 + IAng2;
FinVertAng3 = FVertAng3 + IAng3;
FinVertAng4 = FVertAng4 + IAng4;

%redefine the height of the point
newp2(3) = ds2*tan(FinVertAng2);
newp3(3) = ds3*tan(FinVertAng3);
newp4(3) = ds4*tan(FinVertAng4);

%unit vector of tension
u2 = newp2/norm(newp2);
u3 = newp3/norm(newp3);
u4 = newp4/norm(newp4);

%solve for tensions
N = [u2(1), u3(1), u4(1); u2(2), u3(2), u4(2); u2(3), u3(3), u4(3)];
M = [0;0; W1];
T = inv(N)*M;

%vectors of weight
vw2 = [0, 0, W2];
vw3 = [0, 0, W3];
vw4 = [0, 0, W4];
%tension on the wheel by the rope
TW2 = T(1)*u2;
TW3 = T(2)*u3;
TW4 = T(3)*u4;

%resultant of the weight and tension
R2 = vw2 + TW2;
R3 = vw3 + TW3;
R4 = vw4 + TW4;

%friction about the axle
Fr2 = (r2/r1)*(norm(TW2) - norm(vw2));
Fr3 = (r2/r1)*(norm(TW3) - norm(vw3));
Fr4 = (r2/r1)*(norm(TW4) - norm(vw4));

%coefficient of static friction
mew2 = abs(Fr2)/norm(R2);
mew3 = abs(Fr3)/norm(R3);
mew4 = abs(Fr4)/norm(R4);

fprintf('%d %f %f %fn', index, mew2, mew3, mew4);
end